OSU Graduate Boot Camp: Reflections

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Structure of the course

Two weeks before fall classes began at Oregon State University I ran a week-long boot camp for incoming graduate students. The purpose of the boot camp is to ensure that incoming students have a common base of content knowledge and proof techniques from basic analysis and linear algebra. These subjects were chosen for two reasons: (1) analysis and abstract linear algebra are required courses for incoming graduate students, and (2) the PhD qualifying exams are in analysis and abstract linear algebra. The structure and content of the boot camp were completely up to me (the department never decided on a curriculum before putting the class on offer). Attendance at the boot camp was completely optional, free, and did not have any official credit associated with it.

Of the 23 incoming first-year graduate students, roughly 18 actually attended each day. Background preparation varied wildly among the students. A handful of these students, 3 to 5 maybe, already had master's degrees in mathematics. Other students had engineering backgrounds or otherwise had a less comprehensive math background than the typical graduate student in mathematics. There was a wide age range as well (youngest student was 20, oldest student was maybe 40).

The structure of the course was as follows. Class went from 9:30am to 3:30pm with a break from noon to 1pm for lunch. I assigned (required) readings and exercises to do before class. When feasible I used the guided readings from the excellent TRIUMPHS projects. (These are projects for undergraduate mathematics courses based on primary historical sources: `https://digitalcommons.ursinus.edu/triumphs/`). I asked students to write up their answers to the exercises and bring them to class (though I did not actually make them hand anything in). The morning sessions (9:30am-noon) ran as discussion sessions focused on the readings and exercises related to them. The afternoon sessions (1pm-3:30pm) ran as problem sessions where I posed more sophisticated problems and sometimes introduced new definitions or theorems for them to work with. Because of conflicts with other university-wide orientation events, I only had 4 days of instruction instead of 5. Three of those days were spent on analysis topics and one was spent on linear algebra. For analysis we covered limits, series, convergence, continuity, and compactness. For linear algebra we discussed the geometry of linear transformations, determinants, inner products, differentiation and integration as linear transformations, and the kernel and image subspaces. Each day I tried to end with a qualifying exam problem that was doable but pushed the ideas to their limits.

I did almost no lecturing at all. This was my first foray into teaching a class under the active learning model. It was an excellent experience and I hope to use it in all my future teaching. More information related to the structure of the class can be found on my website: `https://sarahhagen.weebly.com/teaching.html`

Morning Sessions: Teaching with the TRIUMPHS Projects

For the first three days of the boot camp the morning sessions were dedicated to working through the following TRIUMPHS projects:

Day 1: "Investigations into d'Alembert's Definition of Limit" `https://digitalcommons.ursinus.edu/triumphs_analysis/8/`. Instructions for students: *Prior to class, please do exercises 1, 5, 11, and 13 from the reading. Please write out your answers to the exercises and bring them to class. Write down and bring to class the definition of a metric space (taken from your favorite analysis book, or Wikipedia, or whatever). Write down and bring to class the definition of limit superior and limit inferior.*

Day 2: "Bolzano's Definition of Continuity, his Bounded Set Theorem, and an Application to Continuous Functions" `https://digitalcommons.ursinus.edu/triumphs_analysis/9/`. Instructions for students: *Prior to class, please do exercises 1, 3, and 4 from the reading. Please write out your answers to the exercises and bring them to class. Write down and bring to class a statement of the Intermediate Value Theorem, the Mean Value Theorem, and the Extreme Value Theorem. Write down and bring to class a statement of the epsilon-delta definition of continuity. Write down and bring to class a definition of uniform continuity, a definition of Lipschitz continuity, and a definition of absolute continuity.*

Day 3: "Abel and Cauchy on a Rigorous Approach to Infinite Series" [sections 1 and 2 only] `digitalcommons.ursinus.edu/triumphs_analysis/4/`. Instructions for students: *Prior to class, please do exercises 1, 2, 6, 8, and 14 from the reading. Please write out your answers to the exercises and bring them to class. Please write out and bring to class a modern statement of what is known as "Cauchy's convergence test" (or "Cauchy's criterion for convergence"). Review the definition of a Taylor series. Does the Taylor series of an infinitely differentiable function always converge? If it does converge, does it always converge to the function that it is derived from? (The Wikipedia entry on Taylor series can be helpful here).*

On day 1, after doing introductions, I broke up the class into groups of 3-5 and asked them to discuss their answers to Exercise 1. While they discussed I walked around to observe and check-in. As I walked around I chose a student to write up their answer to Exercise 1 on the board. They wrote while the class continued their discussions. Once the exercise was written on the board we discussed it as a class. My contribution was mainly as a facilitator of the discussion. The student sat down and then I asked the students to work on exercises 3 and 4 on their own for a few minutes and then to start discussing with their classmates once they felt that would be productive for them. Again, once discussions started I walked around and chose new students to write on the board their solutions. With the solutions on the board I facilitated a discussion. This became the rhythm of the class as we worked through the rest of the packet. The other two morning activities proceeded similarly.

Starting out the mornings with the TRIUMPHS projects was a great idea (thank you Dr. Pengelley!). It served as an excellent ice-breaker for introducing group work. The students seemed to really enjoy reading and discussing the historical material. We all had a nice laugh at the expense of these famous mathematicians who still made mathematical blunders and used imprecise (and sometimes mystical) language. These discussions were particularly nice because they evened the playing field between the students with greater and lesser experience/mathematical maturity. It also provided a great launching point for the afternoon discussions. It allowed us to explore fundamental concepts and reintroduce modern formulations in a way that maintained the interest of everyone involved (including me!). Discussions were lively and playful. The class had particular fun discussing Bolzano's claim that "it is an unacceptable breach of good method to try to derive truths of pure (or general) mathematics (i.e. arithmetic, algebra, analysis) from considerations which belong to a merely applied (or special) part of it, namely geometry."

The TRIUMPHS packets naturally lent themselves to group work and discussion. Thus they were extremely helpful for setting the classroom norms for active learning. By the time we reached the problem sessions in the afternoon it was second nature to break into groups to discuss and solve problems.

Another virtue of starting out the days with the TRIUMPHS readings is that it provided a natural way to break up the days. Merely doing problems all day can get repetitive and exhausting. It was nice to change gears between the morning and afternoon sessions.

In general, it seemed like the students felt that the TRIUMPHS projects were both fun and easy. We went through the material faster than I had anticipated, which forced me to come up with more challenging (but related) exercises on the fly. This was fun and actually led to some excellent discussions. (I should note that the students found Bolzano's proof of the intermediate value theorem pretty time consuming and a little dizzying to work through.)

Afternoon Sessions: Problem Solving

In the afternoons I had the students work on problems that were related to (or inspired by) the concepts discussed in the morning TRIUMPHS packet. So, for example, on the day we discussed D'Alembert's definition of the limit we used the afternoon to prove general theorems about limits, to find the limits of some famous sequences, and to introduce and work with some extensions of the limit (lim inf and lim sup). I ended the day with a qualifying exam problem that dealt with subadditive sequences. As preparation for the afternoon sessions, I asked students to look up, write down, and bring to class definitions and theorems that we would be using that day. I either asked small questions related to these new concepts or simply asked the students to try to understand what they were saying before coming to class (just so that they would not be coming at the concepts completely cold).

These problem sessions served to introduce the actual background material that the boot camp was supposed to convey. They also served to up the difficulty of the course. (As I mentioned before, the students thoroughly enjoyed, but for the most part were not terribly challenged by, the TRIUMPHS projects). The students seemed to enjoy the harder problems, and ending with a qual problem kept the advanced students engaged.

One thing that did not work as well with the problem sessions is that the students who already held master's degrees knew immediately how to do almost every problem that I gave (except for the qual problems). This, I heard from a couple of students afterwards, made the less well-prepared students feel a little slow or left behind. I actually did not realize until half-way through the boot camp that there were students with master's degrees in the class. (The boot camp is definitely not intended for such students, and I'm not sure why they signed up in the first place.) I think that the other students wanted more time to work on the problems on their own before jumping into discussion, but felt pressured by the more advanced students to give up before they were done productively thinking. Two students mentioned to me after class that they did not do great in group work. This surprised me because they both seemed like they were really engaged in the class. I tried to tease out what wasn't working, and it seems like the super advanced students were at least part of the problem. Another issue is that the classroom we had was small, and so the discussions ended up being pretty loud. I think this made it tough for some students to hear and to think productively during the discussions.

Linear Algebra Day

I did not use a TRIUMPHS project this day only because the current options for linear algebra are limited, and none of the projects that were available were quite right for what I wanted. On this day I gave the students a hodge-podge of readings and a video to watch before class. We spent the morning understanding various geometric interpretations of linear transformations ("row picture", "column picture", and "vector space transformation picture"). We also looked at the geometry of determinants and the dot product and what is happening geometrically when we do Gaussian elimination. Basically, we just drew pictures all morning and had a good time with it. Of course, I followed the group work/ active learning format for this session.

The afternoon was dedicated to applications, problems, and counterexamples in linear algebra. Again, the group work/active learning format was used.

Reflections on Teaching with Active Learning

I hope to never lecture again. This experience of teaching with active learning was eye opening for me. So many things jumped out at me as reasons to prefer active learning to lecturing. First of all, I noticed that students were much more engaged when their classmates presented on the board than when I presented on the board. The students seemed to feel freer to ask "dumb questions" to their classmates than to me. If they didn't understand a line in a proof that the student had written they would just ask them to explain more. However, the few times that I wrote on the board I could feel the room tense up and the students' eyes glaze over (and I pride myself on being an engaging instructor!). The problem is that they felt like what I had to say was gospel, and so they didn't engage as much or question as much. They just accepted what I wrote, copied it down, and then waited for the next piece of information.

The next thing I noticed is that students often came up with solutions very different than what I had in mind. When there were multiple solutions to the same problem I had students write them all up. We then looked them over, compared them, and discussed the virtues and drawbacks of each attempt. This would *never* have happened if I had simply written my own proof on the board. Also, it sometimes happened that there would be a subtle error in a proof that made it to the board (usually I knew in advance that there was an error, and there were a handful of students who were confident enough in themselves that I felt fine having them write something on the board that I knew was wrong). These errors prompted great discussions every time. Usually one of the students would notice the error but maybe not know how to fix it. Often some students would still need convincing that there really was an error in the first place. As a class we would discuss the various nuances, and I pretty much never had to swoop in and resolve the mystery. The students would solve it themselves. At the end of these debates I would spend a minute summarizing the debate and emphasizing the main takeaways. When appropriate I would also place the debate in a broader context or relate it to something else we had done that week. These moments allowed me to share my expertise in a way that would not have come out if I had been lecturing.

This led me to the realization that lecturing is a waste of an instructor's expertise. What is the benefit of having me write down the standard proof of a famous theorem on the board when this is something that the students could find in any textbook? I could perhaps explain the reasoning behind certain moves. But then, if the students have read the proof beforehand, I could still give that explanation, and without wasting time writing out symbols on the board that the students already have in front of them. Even better, I could ask questions or assign exercises that allow the students to discover the subtleties on their own. That is where my experience and knowledge becomes useful. It is in designing the readings, exercises, and discussion questions that best facilitate understanding. It is in answering questions on the fly to help students get beyond some mental block. It is in facilitating thoughtful discussions that bring out the nuances in subtle reasoning. Writing something on the board that the students all have in front of them already is not only a waste of my own expertise, it is a colossal waste of everyone's time.

Teaching with active learning was way more fun than I had anticipated. I have always enjoyed lecturing, but teaching with active learning was more fun, more engaging, less stressful, and ultimately less work than lecturing. The fun was being able to interact with the students on a more personal level. It was also a fun challenge to meet the students where they were and figure out how to get them on board. Writing proofs on the board can be stressful. Subtle errors easily creep in, and when you are writing on the board you don't always see the mistakes that you made (even if you would notice them immediately if someone else was writing on the board). With the model of making students write their answers on the board, the stress of presenting is broken down into small pieces and spread out amongst the whole class. Students also gain experience and confidence presenting their work (something that academics are expected to do frequently). Also, students often enjoy showing off a little bit when they have proved something on their own and they are sometimes quite eager to share.

By the end of the week of boot camp, I trusted the students enough that I could relax a bit on my own preparation. At the beginning of the week I had my own answers to every problem or exercise that I assigned written out in painstaking detail. This was useful as the students were getting accustomed to the format of the class and needed more precise prodding. However, by the end I was no longer worried that no one would be willing to write on the board or that a subtlety would be overlooked if I didn't micromanage. As a result, by the end I was assigning problems that I knew were doable and that I had an idea of how to do, but that I hadn't worked out fully on my own. This made class even more fun for me because I got to think along with the class. It was also good for the students because they got to see more into how a relative expert tackles certain problems and why I can do them faster even though I'm not any smarter than them. I would say things like "I'm not sure how that goes, but when I see something like this it always makes me think of..." So my being slightly less prepared actually allowed students to gain new insights into how to problem solve at the graduate level. (Not that I am advocating being underprepared for class! I am only saying that after a while it was useful to hand the reins over to the class a little more, and that my relaxation of control had its own benefits.)

Things I Would Do Differently Next Time

Overall I would keep much of the boot camp content and structure the same. Now that I have a sense of how much time the readings and problems take, I would choose slightly different exercises to prioritize to make sure that we get to all the important stuff (I felt myself rushing through some of the continuity stuff). One thing that is tough is that I cannot expect too much work from the students outside of class. The standard recommendation of 2 hours of work outside of class for every hour of class time is absurd when class is meeting 5-6 hours a day. This is one reason why I feel like the less well-prepared students felt a little rushed during the problem sessions. Then of course there was the issue of the super advanced students pushing things along faster than much of the class was comfortable with. I'm still not sure how to deal with this problem. Obviously the advanced students felt like they were getting something out of the boot camp since they showed up every day for the entire week. This makes me reluctant to shut them out of the boot camp. It was also nice to have their perspective in class during discussions. In the future I might try to have those students with master's degrees work in their own group and do harder versions of the problems I assign the rest of the class (though I don't love the idea of isolating a subset of the class either). Maybe let the students self-assign into "qual-problem" groups and "background problem" groups for the afternoon sessions... At the very least I will be aware of the fact that there are such advanced students and I will try to make sure that the other students don't feel like they should compare themselves to them.

I would also ask for a larger classroom next time so that the students don't feel like they have to talk over one another to be heard.